The Classical Periphery of Quantum Mechanics – *The Example of Particle Tracks in Detectors*[1]

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Summary of lecture

I will consider regimes of Quantum Mechanics that can be described in classical terms. Such regimes constitute what I call the *"Classical Periphery/Skin of Quantum Mechanics."*
I won't develop the general theory, but illustrate it in a study of *tracks left behind by quantum-mechanical particles* propagating in detectors. These tracks are close to classical particle trajectories.

I will begin my talk with some general comments on the notion of *"events"* in Quantum Mechanics and their role in understanding *"state reduction"*, as manifested in measurements and observations. My discussion is cast in what I have dubbed "ETH-Approach to QM", a presumed cornerstone of Quantum Geometry.

C.G. Darwin

N. Mott

Credits and Contents

- *Thanks are due to:* M. Ballesteros, T. Benoist, N. Crawford, M. Fraas, A. Pizzo, and B. Schubnel for collaborations; *and to:* M. Bauer, Ph. Blanchard, F. Finster, S. Goldstein, B. Kümmerer, C. Paganini, A. Tilloy, and others – for useful discussions.

I gratefully remember many useful encounters and discussions with Detlef.

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1. Comments on the foundations of QM
2. Indirect measurements in QM
3. Particle tracks in detectors – BFF
4. Conclusions

"The interpretation of quantum mechanics (QM) has been dealt with by many authors, and I do not want to discuss it here. I want to deal with more fundamental things" – P.A.M. Dirac

1. Comments on the Foundations of Quantum Mechanics

"... their attempts to see in the very inadequacy of the conventional interpretation of quantum theory a deep physical principle have often led physicists to adopt obscurantist, mystical, positivist, psychical, and other irrational worldviews." (David Deutsch)

This talk touches upon the *foundations of Quantum Mechanics*, which ought to occupy center stage of contemporary theoretical physics. There appear to be lots of "false prophets" talking about this topic, and many contributors prefer to endlessly talk about puzzles and paradoxes and the "weirdness" of Quantum Mechanics – rather than to sit down and try to solve the most pressing open problems.

In fact, I suspect that many physicists, information scientists, philosophers,... would be deeply disappointed if someone actually **solved**, for example, the so-called *"Measurement Problem."* This would deprive them of the pleasure of debates with very little outcome. They therefore prefer to think that it is **impossible** to make decisive progress in answering some of the deep questions about the foundations of QM, and they do not pay attention to people who claim otherwise. – Of course, I disagree with them!

Fundamental questions about QM

"It seems clear that the present quantum mechanics is not in its final form." (P.A.M. Dirac)

In our courses, we tend to describe the QM of a physical system, S, in terms of a *Hilbert space*, $\mathcal{H}_S$, of pure state vectors and a *unitary propagator*, $(U_S(t,s))_{t,s\in\mathbb{R}}$, describing the "time evolution" of its states.

Unfortunately, these data hardly encode *any* interesting information about S that would enable one to draw conclusions about its physical properties, and they give the **erroneous** impression that QM might be a *linear and deterministic theory*.

$\longrightarrow$ Fundamental questions and problems:

1. What do we have to **add** to the usual formalism of QM to arrive at a mathematical structure that, through interpretation, can be given unambiguous physical meaning – *without the intervention* of "observers"? – Let's take Dirac's opinion seriously!

2. Where does the *intrinsic randomness* of QM come from, given the deterministic character of the Schrödinger and Heisenberg equations? How does it differ from classical randomness?

More questions and some claims

In trying to answer these questions one meets further questions:

3. What is an *isolated, but open system* in QM; why is this an important notion? How can one prepare an isolated system in a *prescribed state*?
4. What are *"observables"/physical quantities* and what are *states* of a physical system in QM? What is the *time evolution* of *physical quantities* and of *states* in the **Heisenberg picture**? What is the role of the Schrödinger equation – if any?
5. What are *potential events* & *actual events* in QM? How are *actual events* correlated with **state reduction** (wave-function collapse)?

Some basic definitions and claims:

i. *"Observables"* of a system S are *linear operators* representing physical quantities and acting on a separable Hilbert space.

ii. An *isolated system* S is one that has negligible interactions with its complement, i.e., the rest of the Universe. In the QM of an isolated system S, the *Heisenberg (-picture) time evolution* of "observables" makes perfect sense and does not depend on knowledge of the complement of S.

More claims

Yet, nothing could be farther from the truth than the claim that the Schrödinger equation yields a correct description of the time evolution of states of an isolated system featuring events!

iii. In non-relativistic QM, *potential events* of S are described by partitions of unity by disjoint orthogonal projections. All potential events possibly setting in at time t or later generate a (v.Neumann) algebra, $\mathcal{E}_{\geq t}$. An isolated system is characterized by a co-filtration, $\{\mathcal{E}_{\geq t}\}_{t\in\mathbb{R}}$, of such algebras. A *state* of S at time t is defined to be a **quantum probability measure** (= normal state) on $\mathcal{E}_{\geq t}$.

iv. An *isolated open system* S, i.e., one releasing *actual events* (defined in a precise way), has the property that

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'}, \quad \text{whenever} \quad t' > t.}$$

This expresses the *Principle of Diminishing Potentialities (PDP)*.

(PDP) can be shown to hold in *relativistic quantum theories*. A strong form of it holds in theories with massless modes, such as photons and/or gravitons.

The *ETH* - Approach to QM

v. When combined with *entanglement*, *(PDP)* yields a *stochastic law* for the *time evolution* of *states* that involves **state reduction** (w.f. collapse) governed by *Born's Rule*. $\rightarrow$ It replaces Schrödinger evolution and explains why *"pure" states* can evolve into *"mixed" states*. It yields a probabilistic completion of QM describing *histories* of *actual events* (= certain orth. projections) and of their recordings.

vi. A **projective measurement** amounts to retrieving information about S by recording sequences of *actual events* released by it; (corner stone in a theory of measurements!)

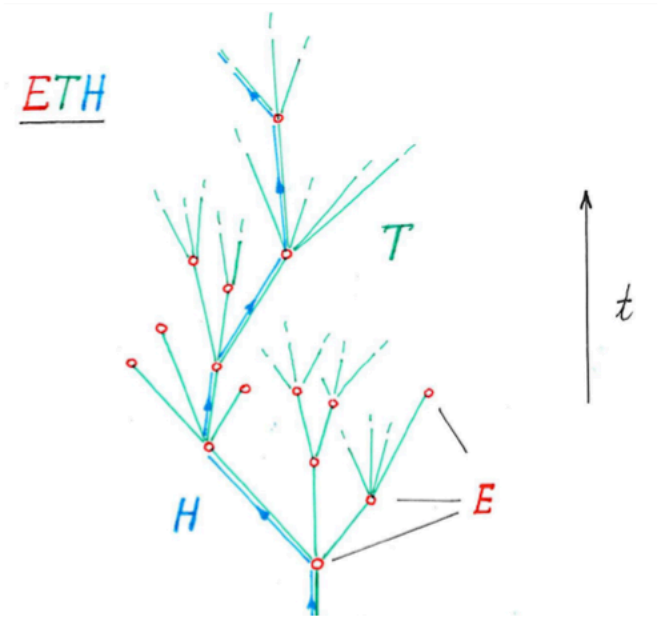
vii. Interventions of *"observers"* are *not* invoked; *actual events* happen spontaneously – $\nexists$ *information- or unitarity paradoxes!* ...

The details underlying these claims form the basis of the so-called *"ETH - Approach to QM"*, where *"ETH"* stands for:

"Events, Trees, and Histories".

Here is a metaphoric picture of the evolution of states according to the *ETH* - Approach:

The evolution of states in the *ETH* - Approach



E: "Events", *T*: "Trees" of possible states, *H*: "Histories" of states

Upshot: In QM, the evolution of *states* of physical systems featuring events can be described in terms of a new kind of *stochastic branching process* (*replacing Schrödinger evolution*) whose non-commutative state space can be described in terms of partitions of unity by disjoint orth. projections (in a universal vN algebra), with branching rules determined by *Born's Rule*.

2. Indirect Measurements in Quantum Mechanics

"Every experiment destroys some of the knowledge of the system which was obtained by previous experiments." (Werner Heisenberg)

In QM, information about a physical system, S, of interest is gained by measurements describable in the *classical periphery* of QM. Often, information on properties of S is gathered by **indirect measurements** involving **probes** (photons, neutrons, atoms, etc.) that interact with S, hence get entangled with $S \rightarrow$ *Plato's allegory of the cave.*

Plato's allegory of the cave as an illustration of indirect acquisition of information in QM

The example of Mott tracks

After their interaction with S the probes are subjected to **projective measurements**, which result from sequences of *actual events* describable within the ETH - Approach to QM. – Because of entanglement, the interaction of a probe with S and its subsequent proj. meas. destroy information on the *incoming state* of S, as intuited by Heisenberg.

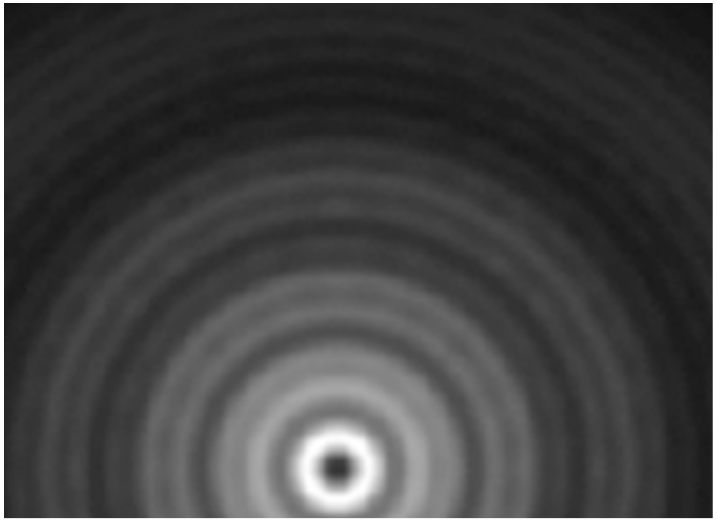
Given a theory of *projective measurements* (e.g., the one provided by the ETH - Approach to QM), the general theory of *indirect measurements*[2] is well developed, and I won't present it here. Instead, I want to illustrate it by explaining how, in QM, classical-looking tracks of particles interacting with the degrees of freedom of a detector (that are then subjected to projective measurements!) can be understood to appear.

History: At the 1927 Solvay conference, in a famous debate with Bohr and Born, the problem of the classical periphery of QM, and in particular the problem of particle tracks, was raised by Einstein. It was later studied by C. G. Darwin and N. Mott, whence the name *"Mott tracks"*.

More recent work on this problem has been done by Blasi et al., O. Steinmann, R. Figari and A.Teta, ... / BBFF, BFF.

[2] Kraus, Maassen and Kümmerer, and others

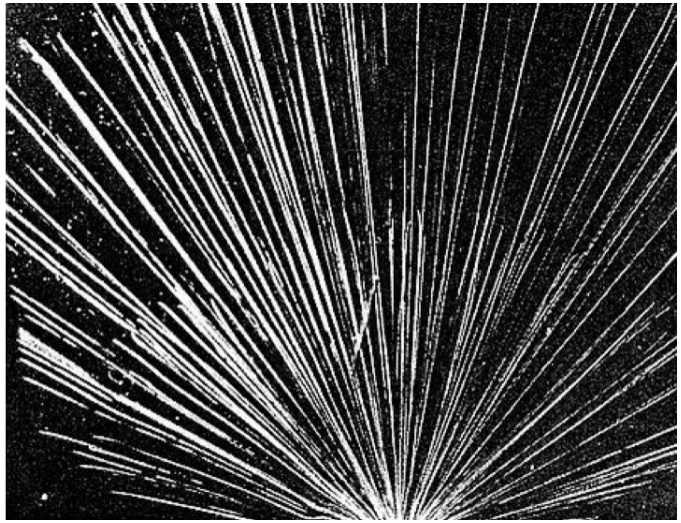
Mott tracks of α-particles emitted by radioactive ball



α-waves in a dark cavity

$\Rightarrow$

Effect of interactions in detector



α-particle tracks in a bubble chamber

Electrons and positrons produced simultaneously from individual gamma rays curl in opposite directions in the magnetic field of a bubble chamber. In the above example the gamma ray has lost some energy to an atomic electron, which leaves the long track, curling left. The gamma rays do not leave tracks in the chamber, as they have no electric charge.

3. Particle Tracks in Detectors – BFF

"The interpretation of quantum mechanics has been dealt with by many authors, and I do not want to discuss it here." (P. A. M. Dirac)

3.1. QM of a charged particle, semi-classical regime

Consider *quantum dynamics* of massive charged particle prepared in an initial state of very high kinetic energy, moving in an ext. magnetic field & periodically illuminated by laser pulses. Scattered light is assumed to hit detectors that click with high probability when a photon arrives. Our goal is to show that, no matter what exactly the initial state of the particle is, the observed approximate particle positions are close to points on the trajectory of a **classical** charged particle of the same mass and charge moving in the same magnetic field.

Hilbert space and Hamiltonian of particle:

$$\mathcal{H} := L^2(\mathbb{R}^3, d^3x) \tag{1}$$

$$H := \frac{1}{2M}\big[P - eA(X)\big]^2 + V(X), \tag{2}$$

where $M = \mathcal{O}(1)$ is the mass and e the electric charge of the particle, $A =$ vector potential of a time-indep., c-number ext. magnetic field B,

Commutation relations, and semi-classical regime

V = ext. potential; P = momentum operator, X = position operator satisfying the commutation relations (CCR)

$$\left[X_i, P_j\right] = i\hbar\delta_{ij}\mathbf{1}, \qquad \left[X_i, X_j\right] = \left[P_i, P_j\right] = 0, \quad i, j = 1, 2, 3. \tag{3}$$

Ψ_t = state of particle at time t, assumed to satisfy the Schrödinger eq.

$$i\hbar\frac{\partial}{\partial t}\Psi_t = H\Psi_t \tag{4}$$

when there is no light scattering (lasers turned off).

Semi-classical regime: Kinetic energy in initial state, Ψ_0, is very large; i.e.,

$$\frac{1}{2M}\langle\Psi_0, \left(P - eA\right)^2\Psi_0\rangle = \mathcal{O}(\varepsilon^{-1}\hbar\,\omega) \tag{5}$$

ω = frequency of laser light. Hence average of *speed* of particle in Ψ_0 is $\mathcal{O}(\varepsilon^{-1/2})$, with $0 < \varepsilon < \varepsilon_0 \ll 1$. $\rightarrow$ Re-scale momentum- and position operators:

$$P =: \varepsilon^{-1/2}\widehat{p}, \quad X =: \varepsilon^{-1/2}\widehat{x}, \quad \text{with} \quad \left[\widehat{x}_i, \widehat{p}_j\right] = i\varepsilon\hbar\,\delta_{ij}\mathbf{1}, \tag{6}$$

other commutators = 0. From now on $\hbar = 1$. *Classical limit:* $\varepsilon \searrow 0$.

Semi-classical regime, ctd.

Choose $A \equiv A_\varepsilon$, $V \equiv V_\varepsilon$ to depend on ε in such a way that

$$A_\varepsilon(\varepsilon^{-1/2}\widehat{x}) \sim \varepsilon^{-1/2} A_0(\widehat{x}), \quad V_\varepsilon(\varepsilon^{-1/2}\widehat{x}) \sim \varepsilon^{-1} V_0(\widehat{x}), \quad \text{as} \quad \varepsilon \searrow 0. \quad (7)$$

In 3D, this choice of A_ε automatically holds for the vector pot. of a uniform magnetic field, $B \in \mathbb{R}^3$, i.e., for $A(X) = \frac{1}{2}(X \wedge B)$; and the formula for V_ε automatically holds for a harmonic potential, $V(X) \propto |X|^2$.

If the relations in (7) hold, the Schrödinger eq. reads

$$i\varepsilon \frac{\partial}{\partial t}\Psi_t = \widehat{H}\Psi_t\,, \quad \text{with} \quad \widehat{H} := \Big[\frac{1}{2M}\big(\widehat{p} - eA_0(\widehat{x})\big)^2 + gV_0(\widehat{x})\Big]. \qquad (8)$$

The semi-classical regime corresponds to $\varepsilon \ll 1$ and initial states, $\Psi_0 \in \mathcal{H}$, with the properties that $\|\Psi_0\|_2 = 1$, and

$$\Delta_{\Psi_0}\widehat{x} \cdot \Delta_{\Psi_0}\widehat{p} = \mathcal{O}(\varepsilon),$$

where

$$\Delta_{\Psi_0} A := \sqrt{\langle \Psi_0, \big(A - \langle A\rangle_{\Psi_0}\big)^2 \Psi_0\rangle} \quad \text{and} \quad \langle A\rangle_{\Psi_0} := \langle \Psi_0, A\Psi_0\rangle.$$

3.2. How QM arises from CM by Weyl quantization

Phase space of classical particle is: $\Gamma := \mathbb{R}^d_x \oplus \mathbb{R}^d_p$ (with $d = 3$), points in Γ are denoted by $\xi, \zeta, \ldots$. Furthermore, $\mathcal{C}(\Gamma)$ = space of bounded, smooth functions on Γ.
The Fourier transform, $\mathcal{F}(a)$, of a function $a \in \mathcal{C}(\Gamma)$ is given by

$$\mathcal{F}(a)(\zeta) \equiv \widetilde{a}(\zeta) := (2\pi)^{-d} \int_\Gamma a(\xi)\, e^{-i\xi\cdot\Omega\,\zeta} \mathrm{d}\xi\,, \quad \text{with} \tag{9}$$

$$\xi := (x, p), \quad \zeta := \begin{pmatrix} \mathfrak{x} \\ \mathfrak{p} \end{pmatrix}, \quad \Omega = \begin{pmatrix} 0 & -\mathbf{1}_d \\ \mathbf{1}_d & 0 \end{pmatrix}.$$

Weyl quantization: $a \mapsto \widehat{a} \equiv \mathrm{Op}_\varepsilon(a)$, of $a \in \mathcal{C}(\Gamma)$ is defined by

$$\widehat{a} \equiv \mathrm{Op}_\varepsilon(a) := (2\pi)^{-d} \int_\Gamma \widetilde{a}(\zeta)\, W(\zeta)\, \mathrm{d}\zeta, \qquad \text{where} \tag{10}$$

$$W(\zeta) \equiv W_\varepsilon(\zeta) := \exp\big[i(\widehat{\xi}\cdot\Omega\zeta)\big], \qquad \zeta := \begin{pmatrix} \mathfrak{x} \\ \mathfrak{p} \end{pmatrix} \text{ and } \widehat{\xi} := (\widehat{x}, \widehat{p})\,, \tag{11}$$

are the *Weyl operators*, and $\widehat{x}$, $\widehat{p}$ are the position- and momentum ops., resp., on $\mathcal{H}$, satisfying (6).

Weyl quantization - ctd.

The Weyl operators $W(\zeta), \zeta \in \Gamma$, are unitary & satisfy *Weyl relations*

$$W(\zeta_1)\, W(\zeta_2) = e^{i\frac{\varepsilon}{2}\,(\zeta_1^t \cdot \Omega\, \zeta_2)}\, W(\zeta_1 + \zeta_2) \tag{12}$$

Note that

$$W(\zeta)^* = W(-\zeta) \quad \text{and} \quad W(0) = \mathbf{1} \quad \Rightarrow \quad \widehat{a}^* = \widehat{a}, \ \text{for } a \text{ real.} \tag{13}$$

Let $\phi_t, t \in \mathbb{R}$, ($t =$ time) denote the *symplectic flow* on Γ generated by classical Hamilton fu., h, corresponding to the Hamiltonian $\widehat{H}$; see (8).

Theorem S-C: If $\widetilde{a}$ and $\widetilde{b}$ are finite (complex) measures on Γ then

$$\widehat{a} \cdot \widehat{b} - \widehat{a \cdot b} = \mathcal{O}(\varepsilon) \quad \Rightarrow \quad [\widehat{a}, \widehat{b}] = i\varepsilon \widehat{\{a, b\}} + \mathcal{O}(\varepsilon) \tag{14}$$

$$e^{i(t\widehat{H}/\varepsilon)}\, \widehat{a}\, e^{-i(t\widehat{H}/\varepsilon)} - \widehat{a \circ \phi_t} = \mathcal{O}(\varepsilon)\,. \tag{15}$$

Eq. (15) is a *Egorov-type theorem*.
For quadratic Hamiltonians (free particles, particles in const. magn. field, harmonic pot., etc.), one has that $e^{i(t\widehat{H}/\varepsilon)}\, \widehat{a}\, e^{-i(t\widehat{H}/\varepsilon)} \equiv \widehat{a \circ \phi_t}$.

3.3. Approximate position measurements

Every $\tau\,(>0)$ seconds, a pulse of light is emitted into a cavity containing the charged particle. Light scattered by the particle is caught by an array of photomultipliers that fire with pos. probability when hit by scattered photons. The firing of photomultipliers represents an *actual event* triggering a state reduction (wave-function collapse): It gives rise to a *proj. measurement* of 3 commuting "observables," $Q = (Q_1, Q_2, Q_3)$, with a measured value $q = (q_1, q_2, q_3)$: a point in $\mathbb{R}^3$ corresponding to the approximate position of the charged particle. – After firing, photomultipliers relax back to their initial state, with a relaxation time $\ll \tau$.

Let ρ = density matrix encoding the state of particle right *before* the firing of the photomultipliers and τ seconds *before* the next light pulse is emitted. The state, $\rho(q)$, of the particle *after* the firing of the photomultipliers corresponding to the point $q \in \mathbb{R}^3$, but just *before* the next light pulse is emitted, is then given by

$$\rho \mapsto \rho(q) := \frac{\Phi_q^*(\rho)}{\mathrm{tr}_{\mathcal{H}}\left[\Phi_q^*(\rho)\right]}, \qquad \text{where}$$

$$\Phi_q^*(\rho) \equiv \Phi_{\varepsilon,q}^*(\rho) := e^{-i(\tau\widehat{H}/\varepsilon)} \sum_{\alpha} \left(\widehat{f}_{q,\alpha}\,\rho\,\widehat{f}_{q,\alpha}^*\right) e^{i(\tau\widehat{H}/\varepsilon)}, \tag{16}$$

Approximate position measurements – ctd.

where every "amplitude" $\widehat{f}_{q,\alpha}$ is the quantization of a smooth function $f_{q,\alpha}(x)$ on $\mathbb{R}^3$ peaked at q, for all $q \in \mathbb{R}^3, \alpha = 1, 2, ..., N\ (< \infty)$, and

$$\sum_{\alpha} \int d\nu(q)\, f_{q,\alpha}(x)^* \cdot f_{q,\alpha}(x) \equiv 1, \quad \text{for some meas. } d\nu \text{ on } \mathbb{R}^3. \tag{17}$$

It is assumed that $|f_{q,\alpha}(x)| \approx 0$ if $|x - q| \gg \lambda$, where λ is the wave length of the light pulses scattered off the particle. (Standard considerations about measurements of scattered photons yield (16) and (17)!)

Identity (17) implies that the map $\rho \mapsto \int d\nu(q)\, \Phi_q^*(\rho)$ is completely positive and trace-preserving. The map $\rho \mapsto \rho(q)$ can be iterated to yield the state $\rho(q_0, q_1, \ldots, q_n)$ of the particle after $n+1$ firings of photo-multipliers:

$$\rho(q_0, q_1, \ldots, q_n) = \frac{\Phi_{q_n}^* \circ \cdots \circ \Phi_{q_0}^*(\rho_0)}{\mathrm{tr}_{\mathcal{H}}\left[\Phi_{q_n}^* \circ \cdots \circ \Phi_{q_0}^*(\rho_0)\right]}, \tag{18}$$

where ρ_0 is the initial state of the particle. With measurement data $\{q_0, q_1, \ldots, q_n\}$ of approx. particle positions we associate the density

$$\mathbb{P}_{\varepsilon,\rho_0}^{(n)}(q_0, q_1, \ldots, q_n) = \mathrm{tr}_{\mathcal{H}}\left[\Phi_{\varepsilon,q_n}^* \circ \cdots \circ \Phi_{\varepsilon,q_0}^*(\rho_0)\right], \tag{19}$$

Probabilities of position-measurement records

which is obviously non-negative. By (16) and (17),

$$\int \prod_{j=0}^{n} d\nu(q_j)\; \mathbb{P}^{(n)}_{\varepsilon,\rho_0}(q_0, q_1, \ldots, q_n) = \mathrm{tr}_{\mathcal{H}}[\rho_0] = 1\,, \tag{20}$$

for an arbitrary density matrix ρ_0 on $\mathcal{H}$. Thus, $\mathbb{P}^{(n)}_{\varepsilon,\rho_0}(q_0, q_1, \ldots, q_n)$ can be interpreted as the *probability density on the space of position-measurements*, $\underline{q}_n := (q_0, q_1, \ldots, q_n)$, conditioned on the initial state ρ_0.

We define a space, $\mathfrak{Q}$, of arbitrarily long sequences of position-measurements, $\underline{q} \equiv \underline{q}_\infty$, by

$$\mathfrak{Q} := \left(\mathbb{R}^3_q\right)^{\times \mathbb{N}_0}$$

By Kolmogorov's extension lemma the measures $\mathbb{P}^{(n)}_{\varepsilon,\rho_0}(\underline{q}_n) \prod_{j=0}^{n} d\nu(q_j)$ are the marginals of a measure $d\mathbb{P}_{\varepsilon,\rho_0}(\underline{q})$ on the space $\mathfrak{Q}$.

Remark on measurements of approximate particle positions:
The initial state, ρ_0, of the particle can be perfectly *spherically symmetric*, and the detector may also be invariant under space rotations. One may then wonder how, after a measurement, the state of the particle may be peaked near a point $0 \neq q \in \mathbb{R}^3$, thus *breaking spherical symmetry.* –

3.4. Measurement records in the semi-classical regime

The point is that the states of photons scattered off the particle are *entangled* with the state of the particle and depend on its position (operator), $\widehat{x}$. If a large number of scattered photons hitting the photomultipliers produce a measurement of the observable Q then the state of the charged particle "collapses" to one localized near a *specific point* $q \in \sigma(Q) = \mathbb{R}^3$; (a phenomenon called *"purification"* first studied by Maassen & Kümmerer; B(C)FFS, et al. $\rightarrow$ *"non-locality of QM"*.)

—

Next, let ϕ_t be the symplectic flow on Γ, as in *Theorem S-C*. We set

$$\xi_j \equiv (x_j, p_j) := \phi_{j\tau}(\xi), \quad \xi \equiv (x, p) = (x_0, p_0) \in \Gamma.$$

Let $d\mu_\varepsilon(\xi)$ be the Wigner distribution of the initial state, $\rho_{\varepsilon,0}$, of the charged particle. We choose a sequence of initial states $\left\{\rho_{\varepsilon,0}\right\}_{\varepsilon>0}$ such that, in the limit as $\varepsilon \searrow 0$, $d\mu_\varepsilon(\xi)$ approaches a *positive probability measure*, $d\mu_0(\xi)$, on phase space Γ.

We now use the results stated in *Theorem S-C* to conclude that

$$\boxed{\mathbb{P}^{(n)}_{\varepsilon,\rho_{0,\varepsilon}}(q_0, q_1, \ldots, q_n) = \mathbb{P}^{(n)}_{\rho_0}(q_0, q_1, \ldots, q_n) + \mathcal{O}(\varepsilon),} \qquad (21)$$

Semi-classical regime – ctd.

where $\mathcal{O}(\varepsilon)$ is an error term (that grows rapidly in n), and

$$\mathbb{P}^{(n)}_{\rho_0}(q_0, q_1, \ldots, q_n) = \int_\Gamma \prod_{j=0}^{n} \Big(\sum_\alpha \left| f_{q_j,\alpha}(x_j) \right|^2 \Big) \mathrm{d}\mu_0(\xi) \tag{22}$$

Let
$$d\lambda(x, q) := \Big[\sum_{\alpha=1}^{N} |f_{q,\alpha}(x)|^2 \Big] d\nu(q).$$

The measures $\mathbb{P}^{(n)}_{\rho_0}(\underline{q}_n) \prod_{j=0}^{n} d\nu(q_j)$ are the marginals of a measure $d\mathbb{P}_{\mu_0}(\underline{q})$ on $\mathfrak{Q}$, given by

$$d\mathbb{P}_{\mu_0}(\underline{q}) = \int_\Gamma d\mu_0(\xi) \prod_{j=0}^{\infty} d\lambda(x_j, q_j), \tag{23}$$

that is *"exchangeable"* (de Finetti).

One expects that *an infinitely long measurement record* $\underline{q} \in \mathrm{supp}\mathbb{P}_{\mu_0} \subset \mathfrak{Q}$ *almost surely determines a unique classical particle trajectory:*

$$\big\{ \xi_j = (x_j, p_j) \big\}_{j=0}^{\infty}, \quad \text{with} \quad \xi_{j+1} = \phi_{(j+1)\tau}(\xi), \ \xi = \tilde{\xi}(\underline{q}) \in \Gamma,$$

3.5. Reconstruction of particle trajectories

where $\xi = \tilde{\xi}(\underline{q})$ is the initial condition of the classical particle trajectory singled out by $\underline{q}$. Let Δ be a measureable subset of $\mathfrak{Q}$, and let

$$\Lambda = \left\{ \xi \,\middle|\, \xi = \tilde{\xi}(\underline{q}), \text{ for } \underline{q} \in \Delta \right\}.$$

be the image of Δ under the map $\tilde{\xi}$. Then

$$\mathbb{P}_{\mu_0}(\Delta) = \mu_0(\Lambda) \qquad (\textit{Born's Rule})$$

The construction of the map

$$\begin{aligned} \tilde{\xi} : \ \mathfrak{Q} &\to \Gamma, \\ \underline{q} &\mapsto \tilde{\xi}(\underline{q}) \in \Gamma \end{aligned} \tag{24}$$

is an "exercise" in statistics. If the Hamilton function h is quadratic in x and p one can use the *Law of Large Numbers* and the *Central Limit Theorem* to accomplish this construction quite easily.
The example of a freely moving particle is particularly simple. We set

$$\mathfrak{p}_j := \frac{M}{\tau}\left(q_j - q_{j-1}\right).$$

The example of a freely moving particle

The *Law of Large Numbers* then implies that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \mathfrak{p}_j = p \equiv \text{momentum coo.of } \tilde{\xi}(\underline{q}).$$

By the *Central Limit Theorem*, the variable

$$\delta\mathfrak{p} := \lim_{N\to\infty} \tfrac{1}{\sqrt{N}} \textstyle\sum_{n=1}^{\infty} \Big(\mathfrak{p}_n - p\Big)$$

is a Gaussian r.v. with mean 0 and a finite variance determined by $d\lambda$, but indep. of $d\mu_0$. – Finally,

$$\frac{1}{N} \sum_{n=1}^{N} \Big(q_n - n\tau \frac{p}{M}\Big) \underset{N\to\infty}{\longrightarrow} x = \text{position coo. of } \tilde{\xi}(\underline{q})\,.$$

By (21), the probability of measnt. data $\underline{q}_n = (q_0, q_1, \ldots, q_n)$, $n < \infty$, *taken from a high-energy particle* (i.e., for ε small) *is close to the probability of* $\underline{q}_n$ *taken from a corresponding classical particle.*

$\Rightarrow$ Thus, highly energetic charged q.m. particle periodically illuminated by laser pulses leave tracks close to trajectories of classical particles.

4. Conclusions

I. It would be interesting to analyze the *quantum corrections*, present whenever $\varepsilon > 0$, to the trajectories reconstructed from the classical measures

$$\left\{\mathbb{P}^{(n)}_{\rho_0}(q_0, q_1, \ldots, q_n)\right\}_{n=0,1,2,\ldots}$$

defined in (22). For $\varepsilon > 0$, the sequ. $\left\{q_n\right\}_{n=0,1,2,\ldots}$ is expected to follow a classical trajectory, up to diffusive noise growing like $\sqrt{n}$.

II. The quantum mechanics of an indirect measurement of the position of a charged particle by bombarding it with a large number of soft photons which then hit detectors is quite well understood within idealized models; (see, e.g., Maassen & Kümmerer, BFFS.)

III. In an earlier paper (BBFF), we have proposed and analyzed an idealized model of a charged particle periodically illuminated by laser pulses that is simple enough to be solved essentially exactly, provided the particle dynamics is quasi-free. The positions of a particle described by this model are shown to line up along Mott tracks even before the semi-classical regime is approached.

Further examples of indirect measurements

However, the exact solution of the model somewhat obscures the basic physical mechanisms giving rise to these tracks. $\longrightarrow$ Thus, we should improve our grasp of the model analyzed in this talk for $\varepsilon > 0$; (*"measurements of non-commuting observables"*!).

IV. There are plenty of examples of how, in QM, information about a physical system, S, is retrieved by having a long sequ. of "boring" *probes* (photons, neutrons, atoms, ...) interact with S, whose states get entangled with state of S and which are then subjected to a crude projective measurement.

An instructive example is the *Haroche-Raymond exp.*:

S : a cavity filled with e.m. radiation initially in a coherent state;
probes : Rydberg atoms with an internal degree of freedom (a "pseudo-spin" $\frac{1}{2}$). When a probe is traversing the cavity its pseudo-spin precesses at a rate depending on the number of photons in S. Afterwards, a projective measurement of a component of its "pseudo-spin" is made.

The experiment of Haroche-Raimond et al.

The measurement record taken from a long sequence of probes (frequency of "spin-up") turns out to determine a precise value of the *number of photons* occupying the cavity (*"purification"*), and *Born's Rule* holds.

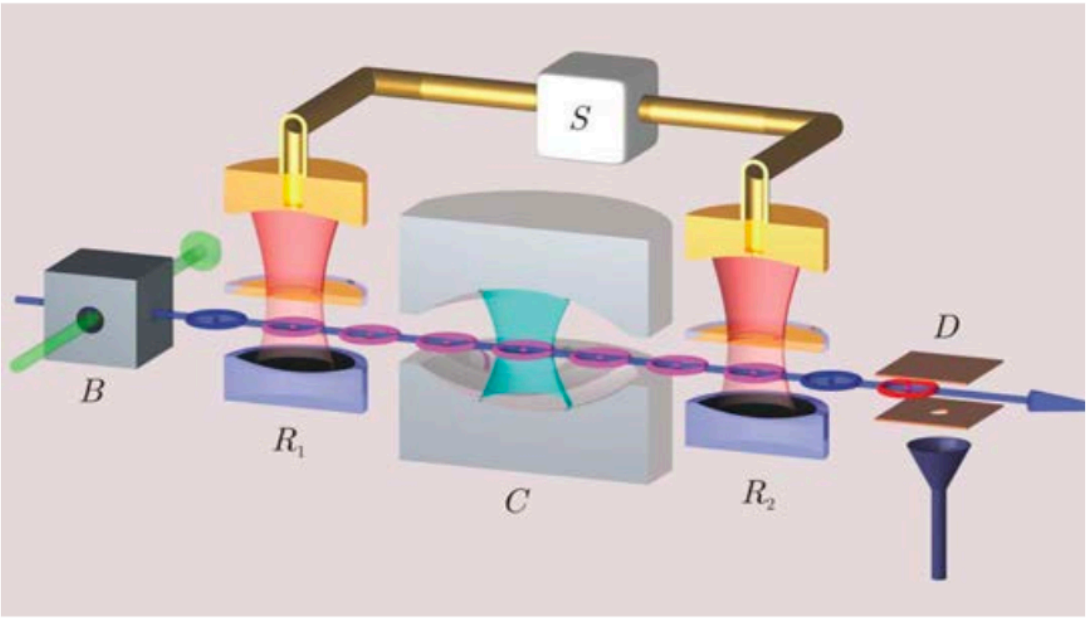


Fig. 4: Experimental setup to study microwave field states with the help of circular Rydberg atoms (see text).

Thank you for your attention !

Epilogue: "Vivre et Survivre" – 50 years later

... depuis fin juillet 1970 je consacre la plus grande partie de mon temps en militant pour le mouvement *Survivre*, fondé en juillet à Montréal. Son but est la lutte pour la survie de l'espèce humaine, et même de la vie tout court, menacée par le déséquilibre écologique croissant causé par une utilisation indiscriminée de la science et de la technologie et par des mécanismes sociaux suicidaires, et menacée également par des conflits militaires liés à la prolifération des appareils militaires et des industries d'armements. ...

Alexandre Grothendieck

Let's take up this struggle again – it matters!